AP CALCULUS FORMULA LIST

Definition of e: $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Absolute Value: $|x| = \begin{cases} x & if \quad x \geq 0 \\ -x & if \quad x < 0 \end{cases}$

Definition of the Derivative:

$f'(x) = \lim_{\Delta x\to\infty} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ $\qquad$ $f'(x) = \lim_{h\to\infty} \frac{f(x+h)-f(x)}{h}$

$f'(a) = \lim_{h\to\infty} \frac{f(a+h)-f(a)}{h}$ $\qquad$ *derivative at* $x = a$

$f'(x) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ $\qquad$ *alternate form*

Definition of Continuity:

f is continuous at c iff:

(1) $f(c)$ is defined

(2) $\lim_{x\to c} f(x)$ exists

(3) $\lim_{x\to c} f(x) = f(c)$

Average Rate of Change of $f(x)$ on $[a, b]$ = $\frac{f(b)-f(b)}{b-a}$

Rolle's Theorem:

If f is continuous on $[a, b]$ and differentiable on (a, b) and if $f(a) = f(b)$, then there exists a number c on (a, b) such that $f'(c) = 0$.

Mean Value Theorem:

If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c on (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Note : Rolle's Theorem is a special case of The Mean Value Theorem

If $f(a) = f(b)$ then $f'(c) = \frac{f(a)-f(b)}{b-a} = \frac{f(a)-f(a)}{b-a} = 0.$

Source: adapted from notes by Nancy Stephenson, presented by Joe Milliet at TCU AP Calculus Institute, July 2005

Intermediate Value Theorem:

If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c) = k$.

Definition of a Critical Number:

Let f be defined at c. If $f'(c) = 0$ or f' is is undefined at c, then c is a critical number of f.

First Derivative Test:

Let c be a critical number of the function f that is continuous on an open interval I containing c. If f is differentiable on I, except possibly at c, then $f(c)$ can be classified as follows.

(1) If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a relative minimum of f.

(2) If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a relative maximum of f.

Second Derivative Test:

Let f be a function such that $f'(c) = 0$ and the second derivative exists on an open interval containing c.

(1) If $f''(c) > 0$, then $f(c)$ is a relative minimum.

(2) If $f''(c) < 0$, then $f(c)$ is a relative maximum.

Definition of Concavity:

Let f be differentiable on an open interval I. The graph of f is concave upward on I if f' is increasing on the interval, and concave downward on I, if f' is decreasing on the interval.

Test for Concavity:

Let f be a function whose second derivative exists on an open interval I.

(1) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

(2) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Definition of an Inflection Point:

A function f has an inflection point at $(c, f(c))$ if

(1) $f''(c) = 0$ or $f''(c)$ does not exist, and if

(2) f changes concavity at $x = c$.

Exponential Growth:

$$\frac{dy}{dt} = ky \qquad y(t) = Ce^{kt}$$

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Derivative of an Inverse Function: $\left(f^{-1}\right)'(x)=\dfrac{1}{f'\left(f^{-1}(x)\right)}$

First Fundamental Theorem of Calculus:

$$\int_a^b f'(x)\,dx=f(b)-f(a)$$

Second Fundamental Theorem of Calculus:

$$\frac{d}{dx}\int_a^x f(t)\,dt=f(x)$$

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt=f\left(g(x)\right)\cdot g'(x) \qquad \textit{Chain Rule Version}$$

The Average Value of a Function:

Average value of $f(x)$ on $[a,\ b]$ $\qquad f_{AVE}=\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

Volume of Revolution:

Volume around a horizontal axis by discs: $\qquad V=\pi\displaystyle\int_a^b\left[r(x)\right]^2dx$

Volume around a horizontal axis by washers: $\qquad V=\pi\displaystyle\int_a^b\left\{\left[R(x)\right]^2-\left[r(x)\right]^2\right\}dx$

Volume of Known Cross-Section:

Cross-sections perpendicular to *x*-axis: $\qquad V=\displaystyle\int_a^b A(x)\,dx$

Position, Velocity, Acceleration:

If an object is moving along a straight line with position function $s(t)$, then

Velocity is: $\qquad v(t)=s'(t)$

Speed is: $\qquad \left|v(t)\right|$

Acceleration is: $\qquad a(t)=v'(t)=s''(t)$

Displacement from $x=a$ to $x=b$ is: $\qquad \mathit{Displacement}=\displaystyle\int_a^b v(t)\,dt$

Note: Displacement is a change in position.

Total Distance traveled from $x=a$ to $x=b$ is: $\qquad \mathit{Total\ Distance}=\displaystyle\int_a^b\left|v(t)\right|\,dt$

TRIGONOMETRIC IDENTITIES

Pythagorean Identities:

$\sin^2 x + \cos^2 x = 1$ $\quad\quad$ $\tan^2 x + 1 = \sec^2 x$ $\quad\quad$ $1 + \cot^2 x = \csc^2 x$

Sum & Difference Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\quad\quad$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angle Identities

$\sin 2x = 2 \sin x \cos x$

$$\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases} \qquad \cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half Angle Identities

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}} \qquad \cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}} \qquad \tan \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

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CALCULUS BC ONLY

Integration by Parts: $\int u\ dv = uv - \int v\ du$

Arc Length of a Function:

For a function $f(x)$ with a continuous derivative on $[a,\ b]$:

Arc Length is: $s = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\,dx$

Area of a Surface of Revolution:

For a function $f(x)$ with a continuous derivative on $[a,\ b]$:

Surface Area is: $S = 2\pi \int_a^b r(x)\sqrt{1+\left[f'(x)\right]^2}\,dx$

Parametric Equations and the Motion of an Object:

Position Vector $= \left(x(t),\ y(t)\right)$

Velocity Vector $= \left(x'(t),\ y'(t)\right)$

Acceleration Vector $= \left(x''(t),\ y''(t)\right)$

Speed (or, magnitude of the velocity vector): $|v(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

Distance traveled from $t=a$ to $t=b$ is: $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Note: The distance traveled by an object along a parametric curve is the same as the arc length of a parametric curve.

Slope (1'st derivative) of curve C at $\left(x(t),\ y(t)\right)$ is: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Second derivative of curve C at $\left(x(t),\ y(t)\right)$ is: $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}$

L'Hôpital's Rule:

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

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Polar Coordinates and Graphs:

For $r = f(\theta)$: $x = r\cos\theta,\quad y = r\sin\theta,\quad r^2 = x^2 + y^2,\quad \tan\theta = \dfrac{y}{x}$

Slope of a polar curve: $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$$

Area inside a polar curve: $$A = \frac{1}{2}\int_\alpha^\beta \left[f(\theta)\right]^2 d\theta = \frac{1}{2}\int_\alpha^\beta r^2\, d\theta$$

Arc length $(*)$: $$s = \int_\alpha^\beta \sqrt{\left[f(\theta)\right]^2 + \left[f'(\theta)\right]^2}\, d\theta = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Surface of Revolution $(*)$:

about the polar axis: $$S = 2\pi\int_\alpha^\beta f(\theta)\sin\theta\sqrt{\left[f(\theta)\right]^2 + \left[f'(\theta)\right]^2}\, d\theta$$

about $\theta = \dfrac{\pi}{2}$: $$S = 2\pi\int_\alpha^\beta f(\theta)\cos\theta\sqrt{\left[f(\theta)\right]^2 + \left[f'(\theta)\right]^2}\, d\theta$$

Euler's Method:

Approximating the particular solution to: $y' = \dfrac{dy}{dx} = F(x,\ y)$

$x_n = x_{n-1} + h \qquad y_n = y_{n-1} + h\cdot F(x_{n-1},\ y_{n-1}) \qquad$ given: $h = \Delta x,\ (x_0,\ y_0)$

Logistic Growth:

$$\frac{dP}{dt} = kP\cdot\left(1 - \frac{P}{L}\right) \qquad P(t) = \frac{L}{1 + Ce^{-kt}}$$

where:
- k is the proportionality constant
- L is the Carrying Capacity
- C is the integration constant

The n'th Taylor Polynomial for f at c:

$$P_n(x) = f(c) + f'(c)\cdot(x-c) + \frac{f''(c)}{2!}\cdot(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}\cdot(x-c)^n$$

The n'th MacLaurin Polynomial for f is the Taylor Polynomial for f when $c = 0$.

$$P_n(x) = f(0) + f'(0)\cdot x + \frac{f''(0)}{2!}\cdot x^2 + \frac{f''(0)}{3!}\cdot x^3 + \cdots + \frac{f^{(n)}(0)}{n!}\cdot x^n$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$$

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