FORMULAS TO KNOW AND MEMORIZE

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

Percent Change = $\frac{New-Old}{Old}$ **x 100**

Proportions $\frac{part}{whole} = \frac{x}{100}$

Average Speed = $\frac{total\ distance}{total\ time}$

Prime Number

Is a number that is divisible by 1 and itself **AND** its factors are DISTINCT (unique or different)
Prime Numbers start with the number 2. 2 is the only even Prime #
Some Prime Numbers 2,3,5,7,11,13,17,19,23,…..

Natural or Counting Numbers how we usually count 1, 2, 3, 4……
Whole Numbers 0, 1,2,3,4,5,6,7……
Integers ……., -5, -4,-3,-2,-1,0,1,2,3,4,5,…….

Rational Numbers can be written as a ratio or fraction of integers in the numerator and denominator
These include terminating decimals, repeating decimals, whole numbers, natural numbers, and integers.

Irrational Numbers decimals that do not repeat but go on forever like e, π, $\sqrt{2}$

Real Numbers the set of numbers that includes Counting/Natural Numbers, Whole Numbers, Integers, Rational, and Irrational Numbers

Imaginary Numbers $i=\sqrt{-1}$ $i^2=-1$

Complex Numbers have a real number component plus an imaginary component written in the form **a+b*i***

Conjugate pairs these have the same two terms but with a different sign between their terms. For example, the conjugate of 3x-6 would be 3x+6

Exponent Rules

$x^n x^m \Leftrightarrow x^{n+m}$ — If bases are the same, when you multiply them ADD the exponents

$\frac{x^n}{x^m} \Leftrightarrow x^{n-m}$ — If bases are the same, when you divide them SUBTRACT the exponents

$(x^n)^m \Leftrightarrow x^{nm}$ — When you have an exponent raised to a higher exponent MULTIPLY the exponents

$x^0 = 1$ — Any base (letter or number) to the 0 power is equal to 1 except (0^0 which is undefined)

$x^{-n} = \frac{1}{x^n}$ — To make a negative exponent positive move the variable from the denominator (bottom) to the numerator (top) or move it from the numerator (top) to the denominator (bottom)

$-x^n = -x^n$ — Leave the negative sign & solve x to the nth power

$(-x)^n$ = (-x) times itself n times

If n is an EVEN exponent, the answer will be a positive (+) number

If n is an ODD exponent, the answer will be a Negative (-) number

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Slope-Intercept Formula

y= mx+b

Horizontal Lines have a slope of 0

Vertical Lines have a slope that is undefined

Parallel lines have the same slope, never meet, no solution meaning no intersection

Perpendicular lines slopes are opposite signs & reciprocals

Slope formula $m=\frac{y_2-y_1}{x_2-x_1}$ with points (x_1, y_1) and (x_2, y_2)

Cartesian Plane

A point is always (x,y)

Domain is the x values of a function or relation
Range is the y values of a function or relation

Independent variable is always on the horizontal axis (x-axis)
Dependent variable is always on the vertical axis (y-axis)

TIME as a variable is considered INDEPENDENT. It should always be on the horizontal (x-axis)

y-intercept is where x=0 (this is somewhere on the y axis)
x-intercept is where y=0 (this will be somewhere on the x-axis). Other names for the x-intercept include **roots, solutions, zeroes**.

Three possible solutions for two lines

- **No solution** – if two lines have the same slope but different y-intercepts, the lines will never intersect. Therefore, there is no point in common, no solution. If you get two different numbers equal like 5=12
- **Infinite Solutions** – two lines have the same slope and same y-intercept when they are both put into the slope-intercept formula y=mx+b If the variables eliminate and you end up with the same numbers equal like 5=5
- **One Solution**- two lines do not have the same slope so they cross at exactly one point

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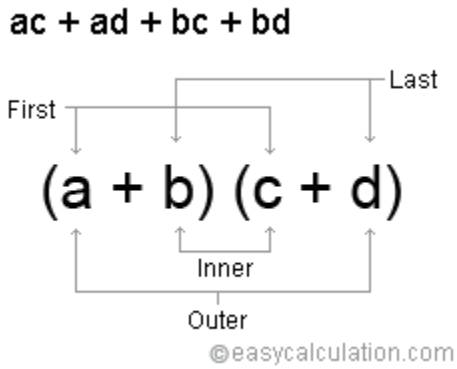
FOIL Method



FUNCTION Definition- A function has every input (x) has only one output (y)
A function will satisfy the vertical line test.

A **one-to-one function** is a special type of function where

Every input (x) has only one output (y) AND every output (y) has only one input (x). It passes **both** the vertical line test and the horizontal line test. If a function is one-to-one, its inverse will be a function.

Inverse functions have symmetry across y=x

Distance Formula d= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint formula M= $(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2})$

Distance=Rate x Time

Density = $\frac{mass}{volume}$

Geometry

$\frac{360}{exterior\ angle}$ = number of sides in a regular polygon

(n-2)180 total degrees in a polygon (where n is the number of sides)

$\frac{(n-3)n}{2}$ = number of diagonals in a polygon

$\frac{(n-2)180}{n}$ = interior angle of a regular polygon

Area of circle πr^2	Circumference of Circle $2\pi r$
Area triangle $\frac{1}{2}bh$	Area rectangle l x w
Area trapezoid $\frac{1}{2}h(b_1+b_2)$	Area parallelogram h x l
Perimeter add all sides	Volume rectangular prism l x w x h
Volume sphere $\frac{4}{3}\pi r^3$	Volume cube s^3
Volume cylinder $\pi r^2 h$	Volume cone $\frac{1}{3}\pi r^2 h$

Property of Triangle with sides a, b, c
(all three must be true)

a+b>c a+c>b b+c>a

Triangle Properties

In a right triangle the largest side is always across from the right angle and is called the hypotenuse. In the Pythagorean Theorem the hypotenuse gets the letter c. The other two sides are considered legs and are given the letters a and b.

Pythagorean Theorem

$a^2+b^2=c^2$

Pythagorean Triples to know

3, 4, 5 5, 12, 13 6, 8, 10 7, 24, 25

In a triangle the smallest side will be across from the smallest angle. The largest side will be across from the largest angle.

Types of triangles

Equilateral-all sides equal, all angles equal (60 degrees)
Isosceles- two sides are equal
Scalene- no sides are equal

Common polygons to know

4 sides quadrilateral 5 sides pentagon 6 sides hexagon
7 sides heptagon 8 sides octagon 9 sides nonagon 10 sides decagon

Types of Angles

Acute less than 90 degrees **Obtuse** greater than 90 degrees **Right** 90 degrees

Complementary add up to 90 degrees
Supplementary angles add up to 180 degrees
Collinear angles lay on the same line and sum to 180
Vertical angles are across from each other and are congruent
Congruent means equal
Similar triangles have the same 3 angles but their sides are in proportion not congruent
Corresponding, Alternate interior and alternate exterior angles are congruent
Same side interior and same side exterior angles are supplementary

Circles

Equation $(x-h)^2 + (y-k)^2 = r^2$ where (h,k) is the center and r is the radius
Diameter of circle = 2 times the radius D=2r

$$\frac{Arc\ length}{Circumference} = \frac{Central\ angle}{360\ degrees} \qquad \frac{Area\ of\ Sector}{Area\ of\ Circle} = \frac{Central\ angle}{360\ degrees}$$

$S=r\phi$ where S is the arc length, r is the radius, ϕ is the central angle in radian degrees

To convert from degrees to radians multiply by $\frac{\pi}{180}$

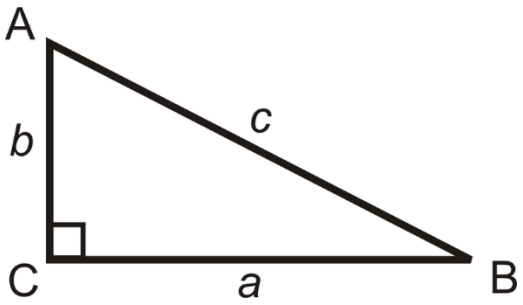
To convert from radians to degrees multiply by $\frac{180}{\pi}$

Trigonometry

SOH sine=$\frac{opposite}{hypotenuse}$ csc=$\frac{1}{sine}$

CAH cosine=$\frac{adjacent}{hypotenuse}$ sec=$\frac{1}{cos}$

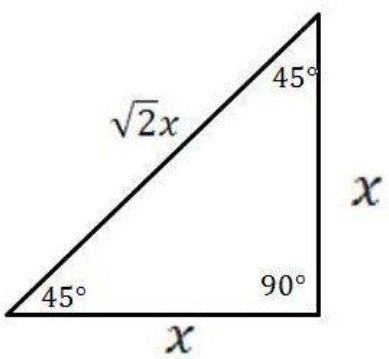
TOA tangent=$\frac{opposite}{adjacent}=\frac{sin}{cos}$ cot=$\frac{1}{tan}=\frac{cos}{sin}$

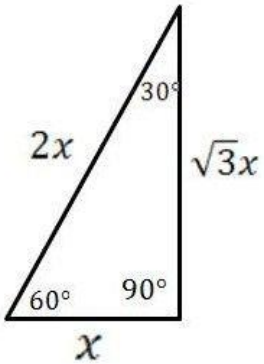


sinA=cosB=$\frac{a}{c}$ **sinB=cosA=$\frac{b}{c}$**

sine and cosine are complementary

Special Right Triangles





Quadratic Formula

$y = Ax^2 + Bx + C$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If $b^2 - 4ac > 0$ - Two real unequal solutions.
If $b^2 - 4ac = 0$ - Repeated real solution.
If $b^2 - 4ac < 0$ - Two complex solutions

Sum of roots = $-\frac{b}{a}$

Product of roots = $\frac{c}{a}$

Parabola

Vertex form $y=a(x-h)^2+k$ where (h,k) is the vertex , when a>0 vertex is concave up, when a<0 the parabola is concave down

A parabola is the set of points that is equidistant between the foci and the directrix.

Exponentials

$y = a(b^x)$ where a is the initial or starting amount

b is the rate. If it is **increasing** or **growth** it is **1+r**

If it is **decreasing** or **decay** it is **1-r**

r is the rate written as a decimal or fraction

Half-life equation

$y = I_0 \left(\frac{1}{2}\right)^{\frac{t}{half-life\ time}}$

Simple Interest

I=PRT
I is the Interest earned
P is the Principal (the money you start out with)
R is the Rate (it must be written as a decimal or fraction)
T is Time and it must be in years (if a problem tells you the time is in months you can convert to years by dividing by 12)

Compound Interest Continuously

$A=Pe^{rt}$
A is the total amount (Principal plus Interest) at the end of a set period
P is the Principal
e is the number on your calculator that roughly equals 2.72

r is the Rate (it must be written as a decimal or fraction)
T is Time and it must be in years (if a problem tells you the time is in months you can convert to years by dividing by 12)

Compound Interest

$A=P(1+\frac{r}{n})^{nt}$

A is the total amount (Principal plus Interest) at the end of a set period
P is the Principal
r is the Rate (it must be written as a decimal or fraction)
T is Time and it must be in years (if a problem tells you the time is in months you can convert to years by dividing by 12)
n is the number of times something compounds per year

annually n=1 monthly n=12 weekly n=52 daily n=365

Logarithm rules

Rule name	Rule
Logarithm product **rule**	$\mathbf{log}_b(x \cdot y) \Leftrightarrow \mathbf{log}_b(x) + \mathbf{log}_b(y)$
Logarithm quotient **rule**	$\mathbf{log}_b(x / y) \Leftrightarrow \mathbf{log}_b(x) - \mathbf{log}_b(y)$
Logarithm power **rule**	$\mathbf{log}_b(x^y) \Leftrightarrow y \cdot \mathbf{log}_b(x)$
	$log_a b = \frac{\log b}{\log a}$

exponent answer answer exponent
$2^{3} = 8$ ⟷ $\log_2 8 = 3$
base base

Common Bases

For any algebraic expressions *S* and *T*, and any positive real number b≠1,
$b^S=b^T$ if and only if S=T

Example: $2^{x+3}=2^5$
$x+3=5$
$x=2$

Power over Root

$\sqrt[n]{x^m}$ **m** is the **power** and **n** is the **root**,

It can be written as $x^{\frac{m}{n}}$

Sequences (sequences are discrete, dots on a graph)

Arithmetic (Linear pattern of dots)

Explicit Formula $a_n=a_1 + d(n-1)$
Where a_1 is the first term, d is the common difference, and n is the number of terms

Recursive Formula $a_1=$
$a_n=a_{n-1} + d$

Geometric (Exponential pattern of dots on a graph)

Explicit Formula $a_n=a_1(r)^{n-1}$
Where a_1 is the first term, r is the common ratio, and n is the number of terms

Recursive Formula $a_1=$
$a_n=a_{n-1}(r)$

Identities

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$a^2-b^2= (a+b)(a-b)$

$(x+a)(x+b)=x^2 + (a + b)x + ab$

$(x + y)^3 = (x + y)(x^2 - xy + y^2)$
$(x - y)^3 = (x - y)(x^2 + xy + y^2)$

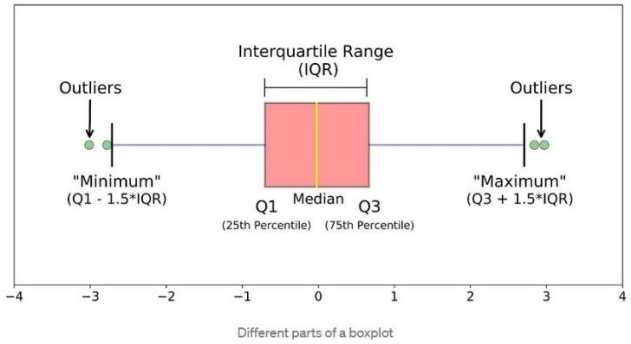
Statistics

Mean (average)=$\frac{sum}{total\ number}$ Mode= the number occurs the most

Median=the number in middle of an odd number of numbers, if there is an even number of numbers take the two in the middle, add them together, divide by 2 to get the median

Range=Highest number minus the lowest number

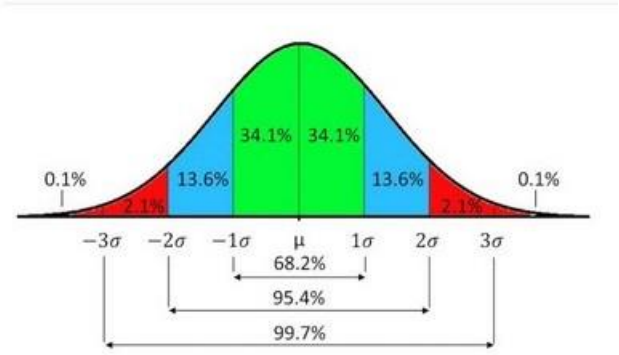
IQR=Interquartile Range Q_3-Q_1



Different parts of a boxplot

Median is the most accurate measure of central tendency because it doesn't get skewed by outliers. Outliers are numbers that are away from where most of the numbers lie.

95% of numbers fall within 2 standard deviations of the mean in a Normal Distribution (Bell-Shaped Curve)



Probability

$$\frac{event}{all\ possible\ outcomes}$$

Probabilities will always be between 0 and 1 (or between 0% and 100%)

P(A or B)= P(A) + P(B) – P(A and B)

P(A or B) is all items in both sets (union)
P(A and B) is the intersection
nP(A)= not P(A) the complement is the set that includes everything not in the P(A)

P(A)+ nP(A)=1 a set plus the complement of its set always equals 1

If events are independent P(A and B)=P(A) times P(B)
P(A|**B**)=**P**(A∩**B**)/**P**(**B**), when **P**(**B**)>0

P of A given B equals P of A and B divided by the Probability of B

When events are mutually exclusive you multiply their probabilities to get the outcome
For example, the probability of getting tails and even on a fair 6 side die would be $\frac{1}{2}x\frac{3}{6}=\frac{1}{4}$= 25%

Sometimes you have probabilities that depend on another. We call those **without replacement**. For example, you have a bag of marbles. 5 are red, 8 blue, 6 yellow. Find the probability of drawing 3 blue marbles without replacement which means you keep the marbles out of the bag with each draw.
The probability of getting a blue on the first draw is $\frac{8}{19}$
The second draw there are now 7 blues left in the bag and 18 marbles left so the probability of getting blue on the second draw is now $\frac{7}{18}$
On the third draw there is now 6 blues left in the bag and 17 total marbles giving a probability on the third draw of $\frac{6}{17}$
The probability of getting three blues in three draws is found by multiplying those three probabilities together
$\frac{8}{19}x\frac{7}{18}x\frac{6}{17}$= 0.05779 This is approximately 5.8%

If we had done the same problem but put the blue marble back in the bag each time, that would be called **with replacement**.
The probability of drawing a blue out of the bag would be the same every time $\frac{8}{19}$
To find the probability of drawing three blues with replacement we would multiply
$\frac{8}{19}x\frac{8}{19}x\frac{8}{19}$ =.07464 which is approximately 7.5%

Intensity

$I=\frac{P}{4\pi r^2}$
I is intensity
P is power
R is radius