

When applying multiple transformations to a function's graph, the order matters. A general order to follow is: 1) horizontal translation (left or right), 2) reflection, stretching, or shrinking, and 3) vertical translation (up or down).

#### 1. Horizontal Translation:

- This involves shifting the entire graph left or right.
- Add or subtract a value inside the function (e.g.,  $f(x - 2)$  would shift the graph right by 2 units).

#### 2. Reflection, Stretching, or Shrinking:

- **Reflection:** This flips the graph over an axis (usually the x-axis or y-axis).
- **Stretching:** This vertically stretches the graph by a factor greater than 1 (e.g.,  $2f(x)$ ) or compresses it by a factor between 0 and 1 (e.g.,  $(1/2)f(x)$ ).
- • **Note:** Horizontal stretching/shrinking is typically done by multiplying the input (x) by a constant (e.g.,  $f(2x)$ ).
- Apply these operations to the function after applying horizontal translation.

#### 3. Vertical Translation:

- This involves shifting the entire graph up or down.
- Add or subtract a value outside the function (e.g.,  $f(x) + 3$  would shift the graph up by 3 units).

#### Example:

Consider the transformation of  $f(x) = x^2$  where we want to shift it left by 2 units, reflect it across the x-axis, and then shift it up by 3 units. The order would be:

1. **Horizontal Translation:**  $f(x + 2)$  (shift left 2 units)
2. **Reflection:**  $-f(x + 2)$  (reflect across the x-axis)
3. **Vertical Translation:**  $-f(x + 2) + 3$  (shift up 3 units)

#### Resources:

- Alamo Colleges: provides a helpful explanation of transformations of functions.
- Missouri Western State University: also covers the order of transformations.
- Khan Academy: offers a visual example of transformations.